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Arithmetic sequence :

eg: 10, 14, 18, 22, 26, ... is an arithmetic sequence.

$\Downarrow$

$a_1, a_2, a_3, a_4, a_5, \ldots$

Determining arithmetic sequence: $a_{n+1} - a_n = k$, where k is a constant $\forall n \geq 1$.

Sum of first n terms $= \frac{(a_1 + a_n)n}{2}$

can view sequence ~~at~~ as set of co-ordinates :

(1, 10) (2, 14) (3, 18) (4, 22) (5, 26) in (x, y) form.

constant slope $\rightarrow$ 4

Can represent this sequence as a formula: $y = 4x + \cancel{10}\ 6$

or ~~$a_n = 4n + 10$~~ $a_n = 4n + 6$ $\rightarrow$ forms a discrete ~~line~~ formula. $\rightarrow n \in \mathbb{N}^*$

Eg: 7, 9, ~~12~~ 11, ...

i) Find a formula for nth term.

ii) Find a_{200}. Find a formula for the sum of 1st n terms

iii) Find the sum of the first 200 terms.

Ans: i) $a_n = mn + b$

$m = 2$ $\rightarrow$ slope $m = a_{n+1} - a_n$ for any $n \in \mathbb{N}^*$

$b = 5$

$\therefore a_n = 2n + 5$

ii) $a_{200} = 2(200) + 5 = 405$

$$\frac{(a_1 + a_n)n}{2} = \frac{(a_1 + 2n + 5)n}{2}$$

$$= \frac{(7 + 5 + 2n)n}{2}$$

$$= \frac{2n^2 + 12n}{2}$$

$$\sum_{i=1}^{n} a_i = n^2 + 6n.$$

iii) $$\sum_{i=1}^{200} a_i = 200^2 + 6(200).$$

$$= 40\,000 + 1200$$

$$= 41\,200.$$

Geometric sequence :

ratio betw. any 2 consecutive numbers / terms is constant.

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots = \frac{a_{n+1}}{a_n} = \text{same constant} = r.$$

Eg: 4, 12, 36, ...

$r = \frac{a_2}{a_1} = 3.$

i) Find a formula for the nth term

ii) Find a formula for the sum of first n terms

Answer: i) $a_n = a_1 r^{n-1} = 4(3)^{n-1}$

eg $a_2 = 4(3)^1$
$= 12.$

$a_5 = 4(3)^4 = 324$

Note: $a_n = a_1 r^{n-1}$ gives discrete exponential formula $\rightarrow n \in N^*$

ii) $\sum_{i=0}^{n} a_i = \frac{a_{n+1} - a_1}{r-1}$

$= \frac{a_1 r^n - a_1}{r-1}$

$\sum_{i=0}^{n} a_i = \frac{a_1(r^n - 1)}{r-1} = \frac{7(3^n - 1)}{2}$

Deriving formula: Suppose $S_n = a_1 + a_1 r + a_1 r^2 + \dots a_1 r^{n-1}$

$\times r$: $rS_n = a_1 r + a_1 r^2 + a_1 r^3 + \dots a_1 r^{n-1} + a_1 r^n$

$rS_n - S_n = a_1 r^n - a_1 = S_n(r-1)$

$\div (r-1)$; $\frac{S_n(r-1)}{r-1} = S_n = \frac{a_1 r^n - a_1}{r-1} = \frac{a_1(r^n - 1)}{r-1}$

Arithmetic – Geometric Sequence:

Eg: Suppose $a_n = 4a_{n-1} - 3a_{n-2}$, $a_1 = 2$
$a_2 = 10.$

Find a formula for a_n.

Note: $a_3 = 4a_2 - 3a_1 = 4(10) - 3(2) = 34$
$a_4 = 4a_3 - 3a_2 = 4(34) - 3(10)$

A sequence like this has a characteristic polynomial $c(a_n)$.

let $a_n = r^n$

substitute; $r^n = 4r^{n-1} - 3r^{n-2}$

$\div r^{n-2}$ (lowest); $r^2 = 4r - 3$

$\Rightarrow r^2 - 4r - 3 = 0$

$r = 1$, $r = 3$

↳ base for geometric sequence

$a_n = c_1(1)^n + c_2(3)^n$
$= c_1 + c_2 3^n$

$n=1$ $2 = c_1 + 3c_2$ $10 = c_1 + 9c_2$

$-6 = -3c_1 - 9c_2$

$4 = -2c_1$

$c_1 = -2$

$c_2 = \frac{4}{3}$

$a_n = -2 + \frac{4}{3}(3)^n$

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Eg: Fibonacci sequence

$a_n = a_{n-2} + a_{n-1} \quad \forall\, n \geq 2$

$a_0 = 1 \quad a_1 = 1 \quad$ (HW).

i) characteristic polynomial:

$r^n = r^{n-2} + r^{n-1}$

$\div\, r^{n-2}: \quad \frac{r^n}{r^{n-2}} = \frac{r^{n-2}}{r^{n-2}} + \frac{r^{n-1}}{r^{n-2}}$

$r^2 = r + \cancel{+}1$

$r^2 - r - 1 = 0$

$r = \frac{1+\sqrt{5}}{2}, \quad r = \frac{1-\sqrt{5}}{2}$

$\therefore \quad a_n = c_1\left(\frac{1+\sqrt{5}}{2}\right)^n + c_2\left(\frac{1\cancel{+}-\sqrt{5}}{2}\right)^n$

let $n = 0$.

$a_0 = c_1 + c_2$

$1 = c_1 + c_2$

$\times -\left(\frac{1+\sqrt{5}}{2}\right)$

let $n = 1$.

$a_1 = c_1\left(\frac{1+\sqrt{5}}{2}\right)^{\cancel{a}} + c_2\left(\frac{1-\sqrt{5}}{2}\right)$

$1 = c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right).$

$\Rightarrow \quad -\frac{1-\sqrt{5}}{2} = -\left(\frac{1+\sqrt{5}}{2}\right)c_1 - \left(\frac{1+\sqrt{5}}{2}\right)c_2$

Add: $\quad -\frac{1-\sqrt{5}}{2} + 1 = -c_2\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right)$

$= -\sqrt{5}\, c_2$

$c_2 = \frac{5-\sqrt{5}}{10}$

$c_1 = \frac{5+\sqrt{5}}{10}$

$\therefore \quad a_n = \frac{5+\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n$